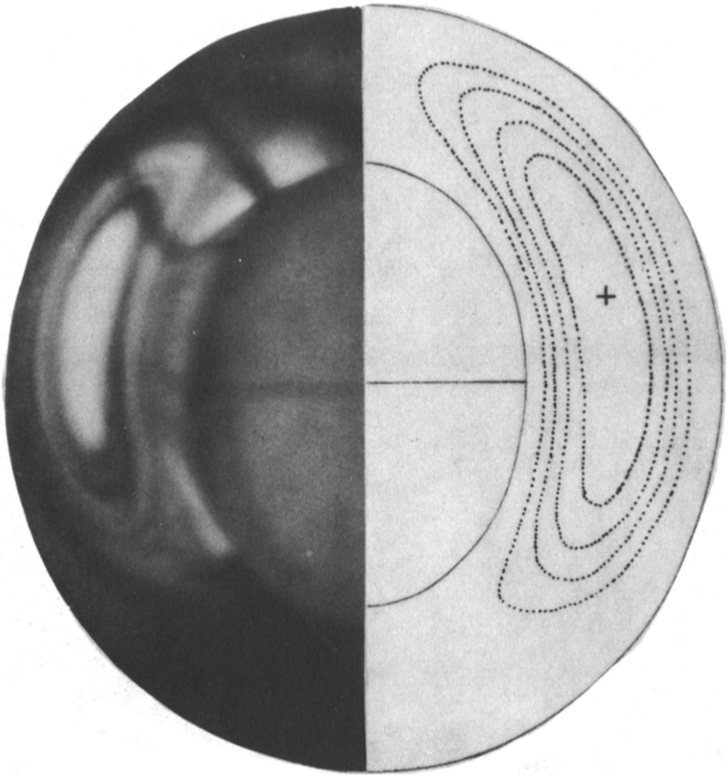

FIG. 5. Comparison between experimental and numerical streamlines at slender configuration for Ra_L = 3.795 × 10^4, Pr = 0.707, a = 20 mm, ε_i = 0.688, ε_o = 0.400.

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TRANSIENT TURBULENT THERMAL CONVECTION IN A POOL OF WATER

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NOMENCLATURE

C_j, constants, $1 \leqslant j \leqslant 5$;
c_p, specific heat at constant pressure;
D, dissipation-rate of turbulent kinetic energy;
g, gravitational acceleration;
k, kinetic energy;
K, thermal conductivity;
l, mixing length;
Nu, Nusselt number;
P, production-rate of turbulent kinetic energy;
Pr, Prandtl number;
q, heat flux;

t, time;
T, temperature;
T_{max}, maximum temperature at any time;
T_1, maximum temperature of the initial distribution, T_{si};
T_2, lowest temperature in the time interval considered;
Z, distance for the surface;
Z^*, defined such that $g\beta(T_1 - T_2)Z^{*3}/\nu\alpha = 1$.

Subscripts
D, dissipation;
H, heat;
i, initial;
k, turbulent kinetic energy;
m, momentum;
s, surface;
si, initial surface;
∞, at large distance from the surface.

Superscripts
$'$, fluctuating component;
$\overline{\ }$, time averaged;
$\bar{\ }$, nondimensional.

Greek letters
α, thermal diffusivity;
β, thermal expansion coefficient;
ε, eddy diffusivity;
ν, kinematic viscosity;
ρ, density.

1. INTRODUCTION

VISKANTA and PARKIN [1] reported experimental results for the cooling of a pool of water in which a stable initial temperature distribution existed because of prior irradiation of the pool, after which the surface of the pool was exposed to a cold plate positioned slightly above the water surface. The history of the heat flow from the surface of the pool was reported, as that was determined from calorimeter measurements on the cold plate. The temperature distribution in the pool was determined by interferometry. Viskanta and Parkin showed that, taking as an initial temperature distribution that which existed at a time 90 s after the placement of the cold plate, when the deduced heat flux from the pool had become relatively constant, a satisfactory prediction of temperature at a time of 120 s could be made by a turbulent model which incorporated a specification of a turbulent eddy diffusivity which depends in a complicated way on the temperature distribution and on the depth within the pool.

As an alternative, there are presented here similar predictions of the temperature distribution in the pool, using one-equation and two-equation models for the turbulence with both conventional and with modified constants.

2. SYSTEM AND EQUATIONS FOR THE ONE-EQUATION MODEL

In the experiment of Viskanta and Parkin, initially the maximum temperature is the initial surface temperature T_{si} and the temperature diminishes with depth to a value T_∞, which exists at a depth at which the prior radiant heating had not altered the temperature of the pool. After cooling of the surface, the surface temperature decreases and the maximum temperature exists below the surface. Below the location of T_{max}, $T > T_i$ because of the downward heat transport that occurs at the later times during the time interval considered.

The temperature distribution is specified by the energy equation written for the assumed stationary water in the pool

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial Z}(\alpha + \varepsilon_H)\frac{\partial T}{\partial Z}. \quad (1)$$

For the solution the eddy diffusivity ε_H must be prescribed. The boundary condition at the surface is

$$\frac{q_0}{\rho c_p} = (\alpha + \varepsilon_H)\frac{\partial T}{\partial Z} \quad (2)$$

with q_0 prescribed.

Also, $T \to T_\infty$ as $Z \to Z_\infty$.

The eddy diffusivity is obtained from a prescribed mixing length and the equation for the kinetic energy of the turbulence. This is

$$\frac{\partial k}{\partial t} = \frac{\partial}{\partial Z}(\nu + \varepsilon_k)\frac{\partial k}{\partial Z} + P - D. \quad (3)$$

For a situation of no mean motion, the production term P is $-\beta g\,\overline{w'T'}$, or $\varepsilon_H \beta g(\partial T/\partial Z)$, effective only when $\partial T/\partial Z$ is positive. The dissipation term D is defined in terms of the kinetic energy of the turbulence and a mixing length

$$D = C_2\frac{k^{3/2}}{l}. \quad (4)$$

Conventionally, when there exists a momentum diffusivity, turbulent Prandtl numbers are defined as

$$Pr_k = \frac{\varepsilon_m}{\varepsilon_k}, \quad Pr_H = \frac{\varepsilon_m}{\varepsilon_H} \quad (5)$$

The magnitude of ε_m is defined as

$$\varepsilon_m = C_1 k^{1/2} l \quad (6)$$

such that

$$\varepsilon_k = \frac{C_1 k^{1/2} l}{Pr_k}, \quad \varepsilon_H = \frac{C_1 k^{1/2} l}{Pr_H}.$$

This much specification, together with a definition of the necessary constants and a prescription about the magnitude of the mixing length, together with boundary conditions for equation (3), enables the solution of the problem. The boundary conditions for equation (3) are taken to be

$$k = 0 \quad \text{as} \quad Z \to \infty \quad \text{and} \quad k = 0 \quad \text{at} \quad Z = 0.$$

The first is definitive, and for the total time interval considered here this is replaced by $\partial k/\partial Z = 0$ at some large enough depth; the second condition implies no agitation at the water surface, a condition which indeed may not apply.

It is convenient but not essential to nondimensionalize the equations. This was done, however, by defining the distance Z^* to be such that $g\beta(T_1 - T_2)(Z^*)^3/\nu\alpha = 1$ where T_1 is selected as the maximum temperature of the initial distribution, T_{si}, and T_2 as temperature lower than any temperature to be realized during the period over which the solution is to be applied (here we have used the surface temperature measured by Viskanta and Parkin [1] at $t = 210$ s). Thus we have

$$\bar{T} = \frac{T - T_2}{T_1 - T_2},$$

$$Z- = \frac{Z}{Z^*},$$

$$\bar{\varepsilon} = \frac{\varepsilon}{\alpha},$$

$$\bar{k} = \frac{kZ^{*2}}{\alpha^2},$$

$$\bar{D} = \frac{DZ^{*4}}{\alpha^3},$$

$$\bar{l} = \frac{l}{Z^*},$$

$$\bar{t} = \frac{t\alpha}{Z^{*2}}$$

and

$$\bar{q}_0 = \frac{q_0 Z^*}{(T_1 - T_2)K}.$$

The equations and boundary conditions then become

$$\frac{\partial \bar{T}}{\partial \bar{t}} = \frac{\partial}{\partial \bar{Z}}(1+\bar{\varepsilon}_H)\frac{\partial \bar{T}}{\partial \bar{Z}} \tag{1a}$$

$$\frac{\partial \bar{k}}{\partial \bar{t}} = \frac{\partial}{\partial \bar{Z}}(Pr+\bar{\varepsilon}_k)\frac{\partial \bar{k}}{\partial \bar{Z}} + Pr\bar{\varepsilon}_H\frac{\partial \bar{T}}{\partial \bar{Z}} - \bar{D} \tag{2a}$$

$$\bar{D} = C_2 \bar{k}^{3/2}/\bar{l} \tag{4a}$$

$$\bar{\varepsilon}_k = \frac{c_1 \bar{k}^{1/2}\bar{l}}{Pr_k} \tag{6a}$$

with the boundary conditions

$$\text{at } \bar{Z} = 0 \qquad \text{as } \bar{Z} \to \infty$$

$$\bar{q}_0 = (1+\bar{\varepsilon}_H)\frac{\partial \bar{T}}{\partial \bar{Z}} \qquad \bar{T} = T_\infty$$

$\bar{k} = 0$ (and therefore $\bar{\varepsilon}_H$ is zero) $\qquad \bar{k} = 0.$

The mixing length was assumed to be

$$\bar{l} = C_3 \bar{Z} \quad 0 < \bar{Z} \leqslant \bar{Z}(\bar{T}_{\max})$$

$$\frac{\bar{l}(\bar{T}_{\max}) - \bar{l}}{\bar{l}(\bar{T}_{\max})} = \frac{\bar{Z} - \bar{Z}(\bar{T}_{\max})}{\bar{Z}_\infty - \bar{Z}(\bar{T}_{\max})} \quad \bar{Z}(\bar{T}_{\max}) < \bar{Z} \leqslant \bar{Z}_\infty. \tag{7}$$

Here $\bar{Z}_\infty$ is the depth at which $\bar{l} = 0$. On this basis the choice for $\bar{Z}_\infty$ is somewhat uncertain. It probably would best be taken as the depth at which the ratio $\bar{T} - \bar{T}_\infty/\bar{T}_1 - \bar{T}_\infty$ attains some relatively small value. This was not done in the present numerical solution, as is explained later.

3. SOLUTION

The solution was done numerically by an explicit finite difference scheme. Before discretization, the transport terms were written in the forms

$$\frac{\partial}{\partial \bar{Z}}(1+\bar{\varepsilon}_H)\frac{\partial \bar{T}}{\partial \bar{Z}} = (1+\bar{\varepsilon}_H)\frac{\partial^2 \bar{T}}{\partial \bar{Z}^2} + \frac{\partial \bar{\varepsilon}_H}{\partial \bar{Z}}\frac{\partial \bar{T}}{\partial \bar{Z}}.$$

This modification was necessary because of the large spatial variation of the eddy diffusivity.

The initial condition for the energy equation was the experimental temperature at $t = 0$, the moment of the exposure of the pool to the cold plate. The calculation was carried out to the time 210 s, the last for which a temperature profile was reported. The problem parameters were set as T_1 = the surface temperature at $t = 0$, 22.556°C and T_2 = the surface temperature at t = 210 s, 19.941°C. Using the properties of water at 20°C there is obtained $Z^* = 0.0145$ cm.

The surface heat flux reported by Viskanta and Parkin was approximated as

$$\bar{q}_0 = 1.16 \times 10^{-5}\,\bar{t} \quad 0 < \bar{t} < 202, \quad 0 < t < 30\text{ s}$$

$$= 2.34 \times 10^{-3} + 9.73 \times 10^{-5}(\bar{t} - 202) \quad 202 < \bar{t} < 403, \quad 30 < t < 60\text{ s}$$

$$= 2.20 \times 10^{-2} + 3.24 \times 10^{-6}(\bar{t} - 403), \quad \bar{t} > 403, \quad t > 60\text{ s}.$$

The initial kinetic energy was taken to be zero.

With the initial condition specified, the problem can be solved when the constants are specified. The values used were

C_1	C_2	C_3	Pr_H	Pr_k
0.5	0.125	0.40	0.59	1.0
0.5	0.125	0.10	0.59	1.0 .

Constants C_1 and C_2 are the values commonly used. They lead to $l = 0.4Z$ when in a flow system the production is taken equal to the dissipation. The value of 0.59 for Pr_H can be considered typical of conditions far removed from fixed boundary. The value of 1.0 for Pr_k is lower than the value of $Pr_k = 1.7$ usually associated with shear flows.

4. RESULTS FOR THE ONE-EQUATION MODEL

Using the initial condition as specified in Section 3, and $C_3 = 0.10$, there were obtained the results shown on Figs. 1 and 2. Figure 1 gives $\bar{T}$ as a function of $\bar{Z}$, with auxiliary scales showing the temperature in °C and the depth in cm. The solid curves show the temperature distribution measured by Viskanta and Parkin for various values of 't' in seconds. The points show the prediction of the one-equation model for these times, and the predictions are always higher than the experimental values, the greatest deviation being for 120 s, where the greatest difference is 0.30°C. Implicitly, the predicted eddy diffusivity is too large.

Figure 1 shows by dashed curves the prediction made by Viskanta and Parkin by the method of Section 4. For it they took as the initial temperature distribution the experimental values at t = 90 s; their predicted temperatures are lower than the experimental values. Generally, and somewhat closer to them, are the present results for the one-equation model.

Figure 2 shows the eddy diffusivity distribution, the points being those for the one-equation model and the curves the predictions from Viskanta and Parkin. The latter are much greater and this accounts for the trend of the temperature predictions for the two models as they are shown on Fig. 1.

5. TWO-EQUATION MODEL

As a matter of interest a two-equation model was also applied, to eliminate the need for a separate specification of the mixing length, at the expense of the need for the assumption of additional constants. The two-equation model consists of the equation for turbulent kinetic energy and the equation for dissipation. The equation for dissipation function 'D' for the situations of no means velocities as given by Launder [2] is

$$\frac{\partial \bar{D}}{\partial \bar{t}} = \frac{\partial}{\partial \bar{Z}}(Pr+\bar{\varepsilon}_D)\frac{\partial \bar{D}}{\partial \bar{Z}} + C_5 \bar{P}\frac{\bar{D}}{\bar{k}} - C_6\frac{\bar{D}^2}{\bar{k}} \tag{8}$$

and with $\bar{D}$ so-determined, the eddy diffusivity for kinetic energy is

$$\bar{\varepsilon}_k = \frac{C_1 C_2 \bar{k}^2}{Pr_k \bar{D}}$$

and

$$\bar{\varepsilon}_D = \frac{C_1 C_2 \bar{k}^2}{Pr_D \bar{D}}. \tag{9}$$

The value of $\bar{D}$ was taken to be zero at the surface and

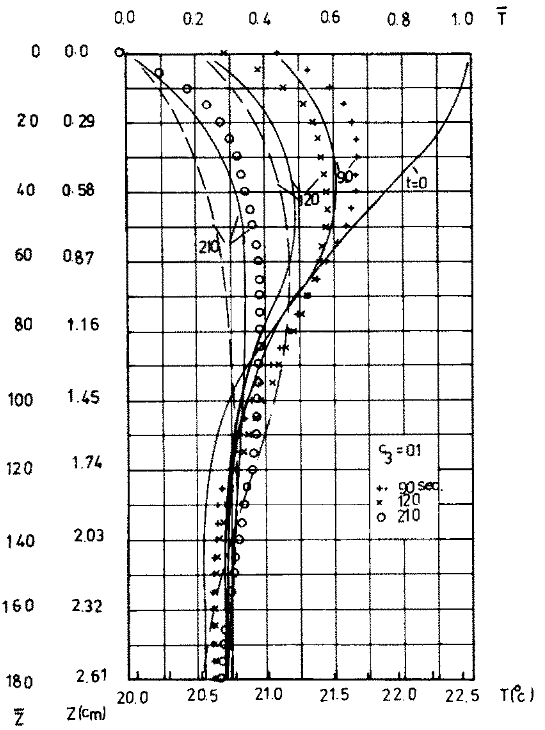

FIG. 1. Temperatures predicted by the one-equation model. Solid curves are the data of Viskanta and Parkin [1]. Dashed curves are the prediction of Viskanta and Parkin. The points, identified on the figure, show the predictions for $C_3 = 0.10$.

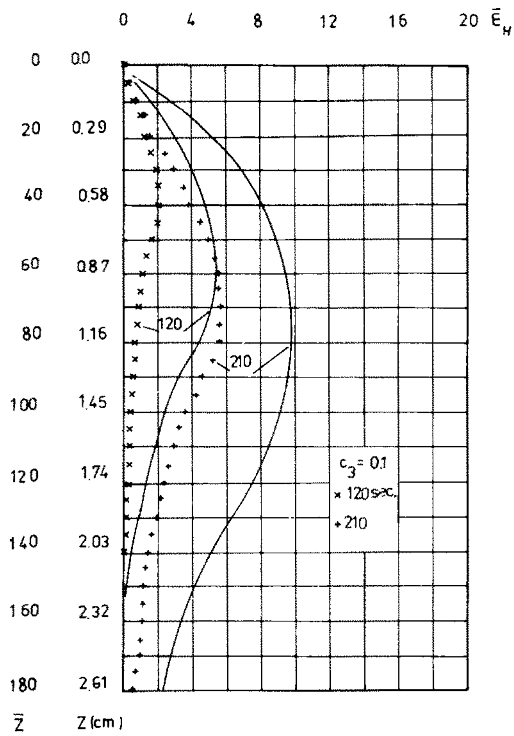

FIG. 2. Eddy diffusivities. Solid curves represent the eddy diffusivities as calculated by Viskanta and Parkin [1]. Points represent the eddy diffusivities obtained in the present calculations.

$\partial\bar{D}/\partial\bar{Z} = 0$ at the greatest depth. Initially, at $t = 0$, $\bar{D}$ should be zero but a small value was assigned thereto to avoid the singularities associated with equation (11).

The additional constants, beyond those needed for the one-equation model, are Pr_D, C_5 and C_6. The values used here are those that have resulted in the relatively successful prediction of steady-state temperature and turbulent kinetic energy distribution (Kaviany [3]) as measured by Deardorff and Willis [4]. These values are modifications of those specified by Launder [2], and the complete set is given by the first row of the following table. The second row gives constants specified by Behnia and Viskanta [5] for a successful prediction for the transient temperature distributions in a pool of water heated from below.

Figure 3 gives the predicted temperature distribution in the same format as Fig. 1. The distribution for 90 s gives temperatures slightly above the experimental values; at 120 s the excess is substantial, at 210 s there is a considerable departure for $\bar{Z} < 40$ and there is the implication of an excessive eddy diffusivity near the surface. The eddy diffusivity was too large because the dissipation was too small. A decrease in the value of C_5 decreases the dissipation, thus the use of a lower value of C_5, such as used by Viskanta and Behnia, would not help resolve the discrepancy evident in Fig. 3.

As a trial, $C_5 = 1.55$ was used, and the resulting eddy diffusivity became smaller. The results for $C_5 = 1.55$ are also shown in Fig. 3 only for the time of 210 s. The value of C_6 was kept the same as before, i.e. 1.9. The temperatures so predicted are closer to the experimental values near the surface but depart more near the maximum temperature.

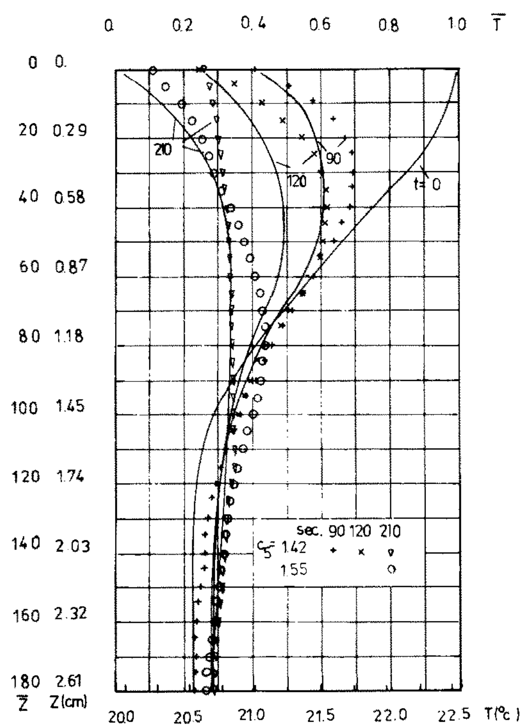

FIG. 3. Temperatures predicted by the two-equation model. Solid curves represent the experimental results of Viskanta and Parkin [1]. Points give the results of the present calculation for $t = 90$, 120 and 210 s for $C_5 = 1.42$ and for 210 s for $C_5 = 1.55$.

6. CONCLUSION

A single equation model of the turbulence, together with a rather arbitrary specification of the distribution of a mixing length, has been shown to give a relatively adequate prediction of the transient temperature history in a pool of water cooled at its upper surface. The predictions are comparable to those made by Viskanta and Parkin by the use of a more empirical specification of the eddy diffusivity itself.

A two-equation model was applied to eliminate the need for the arbitrary specification of the mixing length. With it, and the constants initially chosen, the predicted temperatures are for short times about as good as those obtained from the one-equation model, but the relatively fast development of the eddy diffusivities makes the predictions for longer times inferior to those obtained from the one-equation model. An increase in the value of the constant in the dissipation term did reduce the eddy diffusivity to a more acceptable magnitude but its distribution was such that the temperature distribution was not in good agreement with the experiment except near the surface.

REFERENCES

1. M. Behnia and R. Viskanta, Laboratory study of flow and thermal structure in heated and/or cooled layers of water, Purdue University, Water Resources Research Center, West Lafayette, Indiana. Technical Report No. 123 (1979).
2. B. E. Launder, On the effect of a gravitational field on the turbulent transport of heat and momentum, Imperial College of Science and Technology, Department of Mechanical Engineering. HTS/74/90 (1974).
3. M. Kaviany, Turbulent generation between horizontal plates, Ph.D. Thesis, Mechanical Engineering, University of California, Berkeley (1979).
4. J. W. Deardorff and G. E. Willis, Investigation of turbulent thermal convection between horizontal plates, *J. Fluid Mech.* **28**, 675–704 (1967).
5. R. Viskanta and J. R. Parkin, Laboratory study of unsteady energy transfer in surface layer of stratified water, *J. Water Resource Res.* **12**, 1277–1285 (1976).
6. K. H. Ng and D. B. Spalding, Turbulent model for boundary layer near walls, *Physics Fluids*, **15**, 20–30 (1972).